Deconfining Phase Transition in 2+1 D: the Georgi-Glashow Model.

Gerald Dunne[a,1], Ian I. Kogan[b,2], Alex Kovner[b,3], Bayram Tekin[b,4]

[a] *Department of Physics, University of Connecticut, Storrs CT 06269, USA*

[b] *Theoretical Physics, University of Oxford, 1 Keble Road, Oxford, OX1 3NP, UK*

Abstract

We analyze the finite temperature deconfining phase transition in 2+1 dimensional Georgi-Glashow model. We show explicitly that the transition is due to the restoration of the magnetic Z_2 symmetry and that it is in the Ising universality class. We find that neglecting effects of the charged W bosons leads to incorrect predictions for the value of the critical temperature and the universality class of the transition, as well as for various correlation functions in the high temperature phase. We derive the effective action for the Polyakov loop in the high temperature phase and calculate the correlation functions of magnetic vortex operators.

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[1] dunne@phys.uconn.edu
[2] kogan@thphys.ox.ac.uk
[3] kovner@thphys.ox.ac.uk
[4] tekin@thphys.ox.ac.uk

1 Introduction

The deconfining phase transition in QCD has been a subject of numerous studies in recent years. Some aspects of the physics of the high temperature phase appear to be perturbative and can be studied in a controlled way at asymptotically high temperatures. However in the phase transition region itself the QCD coupling is large and the physics is dominated by the nonperturbative soft sector. Analytic understanding of this region is therefore extremely difficult. It thus seems useful to find a simpler toy model where a similar deconfining phase transition is within the weak coupling regime and can be studied analytically. Optimistically one can hope to learn some lessons from such a model which could be of use in our understanding of real life QCD.

A model that is very suitable for this purpose is the Georgi-Glashow model in 2+1 dimensions - Yang Mills theory with a Higgs field in the adjoint representation. Over twenty years ago Polyakov [1] showed that in this theory the instantons (monopoles) dominate the nonperturbative dynamics and disorder the Higgs vacuum leading to a linear confinement of electric charges even at arbitrarily weak coupling. This was the first demonstration of the dual Meissner effect (or dual superconductivity) in a non-Abelian gauge theory where monopoles condense and expel electric fields from the vacuum, providing an example of the enigmatic confinement phenomenon in controllable circumstances. At zero temperature the 3D Yang-Mills-Higgs system is therefore always in the confining phase, even though classically the confining and Higgs regimes appear to be separated by a phase transition.

In this paper we will study the theory at finite temperature with particular emphasis on the deconfining phase transition. We note that a similar study appeared in the literature several years ago [2]. Our results are both qualitatively and quantitatively different from those of [2]. In particular we show that the deconfining phase transition is primarily due to the plasma effects of charged excitations (W bosons), an effect that was neglected completely in the analysis of [2]. As a result our predictions for both the value of the critical temperature and the universality class of the phase transition are different.

The structure of this paper is the following. In section 2 we review the model and discuss the two complementary views of the confinement mechanism: one based on the monopole plasma [1] and the other based on vortex condensation [3, 4]. In section 3 we discuss the naive expectations for the deconfinement phase transition which follow from these two approaches. We show that these expectations are different and explain why the monopole plasma intuition (the basis of the approach of [2]) should be refined in order to discuss the phase transition properly. In section 4 we describe the renormalization group analysis of the phase transition. We determine the value of the critical temperature, the universality class and discuss the crucial role played by the electrically charged particles. In section 5 we relate our analysis to the standard tool used in the high temperature phase - the effective potential for the Polyakov line. We derive this effective potential in the deconfining phase and show with its help explicitly that the correlation function of vortex operators at $T > T_c$ decreases exponentially at large distances and thus that the magnetic Z_2 symmetry is restored. Finally in section 6 we discuss briefly our results.

2 The model

We consider the $SU(2)$ gauge theory with a scalar field in the adjoint representation in 2+1 dimensions.

$$S = -\frac{1}{2g^2}\int d^3x \mathrm{tr}\,(F_{\mu\nu}F^{\mu\nu}) + \int d^3x \left[\frac{1}{2}(D_\mu h^a)^2 + \frac{\lambda}{4}(h^a h^a - v^2)^2\right] \tag{1}$$

We adopt the notation $A_\mu = \frac{i}{2}A^a_\mu \tau^a$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$, $h = \frac{i}{2}h^a\tau^a$, and $D_\mu h = \partial_\mu h + [A_\mu, h]$ and the trace is normalized as $\mathrm{tr}(\tau^a\tau^b) = 2\delta^{ab}$.

We will be interested throughout this paper in the weakly coupled regime $v \gg g^2$. In this regime perturbatively the gauge group is broken to $U(1)$ by the large expectation value of the Higgs field. The photon associated with the unbroken subgroup is massless whereas the Higgs and the other two gauge bosons $W^\pm$ are heavy with the masses

$$M_H^2 = 2\lambda v^2, \qquad M_W^2 = g^2 v^2. \tag{2}$$

Thus perturbatively the theory behaves very much like electrodynamics with spin one charged matter. The nonperturbative effects however are very important at large distances. Their effect is that the photon acquires finite mass and the charged $W^\pm$ become linearly confined at large distances with nonperturbatively small string tension.

2.1 The monopole plasma

The nonperturbative configurations that dominate the generating functional in the weakly coupled regime are solutions to the classical Euclidean equations of motion,

$$D_\mu F^{\mu\nu} = [h, D^\nu h] \quad ,$$
$$D_\mu D^\mu h = -\lambda(h^a h^a - v^2)h \quad . \tag{3}$$

The perturbative vacuum manifold is $SO(3)/U(1)$ and this has a nonzero second homotopy group, $\Pi_2(SO(3)/U(1)) = Z$. The theory thus allows topologically non-trivial solutions. Polyakov showed [1] that, in the vacuum only far separated monopole configurations are relevant. The unit charge configuration is of the well known 't Hooft-Polyakov monopole form,

$$h^a(\vec{x}) = \hat{x}^a h(r)$$
$$A_\mu^a(\vec{x}) = \frac{1}{r}\left[\epsilon_{a\mu\nu}\hat{x}^\nu(1-\phi_1) + \delta^{a\mu}\phi_2 + (rA - \phi_2)\hat{x}^a\hat{x}_\mu\right] \tag{4}$$

where $\hat{x}^a = x^a/r$. The non-Abelian field strength is

$$F^a_{\mu\nu} = \frac{1}{r^2}\epsilon_{\mu\nu b}\hat{x}^a\hat{x}^b(\phi_1^2+\phi_2^2-1) + \frac{1}{r}(\epsilon_{a\mu\nu} - \epsilon_{\mu\nu b}\hat{x}^a\hat{x}^b)(\phi_1' + A\phi_2)$$
$$+(\delta^{a\nu}\hat{x}^\mu - \delta^{a\mu}\hat{x}^\nu)\frac{1}{r}(\phi_2' - A\phi_1), \tag{5}$$

where prime denotes differentiation with respect to 3D radial coordinate r. The finiteness of the action (in the gauge $A = \phi_2 = 0$) requires $h(0) = 0$, $h(\infty) = v$, $\phi_1(0) = 1$, and $\phi_1(\infty) = 0$.

The monopole has a core size of the order of the vector meson mass. Outside the monopole core, the theory is Abelian and one can define the Abelian field strength as,

$$F_{\mu\nu} = \frac{h^a}{h}F^a_{\mu\nu} - \frac{1}{h^3}\epsilon_{abc}h^a(D_\mu h)^b(D_\nu h)^c. \tag{6}$$

The monopoles (instantons) interact via a Coulomb potential in the Euclidean three dimensional sense, and the partition function of the Coulomb gas can be written as a path integral over the "dual photon field" ϕ. The effective Lagrangian for the field ϕ reads

$$\mathcal{L}_{eff} = \frac{g^2}{32\pi^2}(\partial_\mu\phi)^2 + \frac{M^2g^2}{16\pi^2}\cos\phi. \tag{7}$$

The dynamically generated photon mass, due to the Debye screening of the monopoles, is given in terms of the fugacity of the monopoles as

$$M^2 = \frac{16\pi^2\xi}{g^2}, \qquad \xi = \text{constant}\frac{M_W^{7/2}}{g} e^{-\frac{4\pi M_W}{g^2}\epsilon(\frac{M_H}{M_W})}. \tag{8}$$

$\epsilon(\frac{M_H}{M_W})$ is such that $1 \leq \epsilon \leq 1.787$ [5], and $\epsilon(\infty) = 1$. The field ϕ is by far the lightest excitation in the theory and thus eq.(7) is the low energy effective Lagrangian of the theory valid below the energy scale M_W. The fundamental Wilson loop calculated in this effective theory [1] has an area law fall off, indicating confinement.

2.2 The vortex condensation

A complementary view of confinement in this theory is the spontaneous breaking of the magnetic Z_2 symmetry due to the condensation of magnetic vortices[3].

Ignoring monopole effects, the model has the magnetic $U(1)$ symmetry generated by the Abelian magnetic field. The conserved current is the dual field strength

$$\tilde{F}^\mu = \frac{1}{4}\epsilon_{\mu\nu\lambda}F_{\nu\lambda}. \tag{9}$$

As discussed in detail in [4], due to the monopole contributions this symmetry is anomalous, and only the Z_2 subgroup is conserved. The non-anomalous Z_2 magnetic symmetry group is generated [8] by the large spatial Wilson loop which encloses the system

$$W(C \to \infty) = \exp\frac{i}{2}\int d^2x B(x) \tag{10}$$

The order parameter for this Z_2 transformation is the operator that creates an elementary magnetic vortex of flux $2\pi/g$ [3, 4]

$$V(x) = \exp\frac{2\pi i}{g}\int_C \epsilon_{ij}\frac{h^a}{|h|}E^a_j(x). \tag{11}$$

This operator is local, gauge invariant and Lorentz scalar. Together with the spatial Wilson loop it forms the order-disorder algebra

$$W(C \to \infty)V(x) = -V(x)W(C \to \infty) \tag{12}$$

It can be shown [4] that the vortex operator condenses in the vacuum and its expectation value is determined by the gauge coupling constant $< V >^2 = g^2/8\pi^2$. The low energy theory from this point of view is given by the Z_2 invariant theory of the vortex field V

$$\mathcal{L}_{\rm eff} = \partial_\mu V\, \partial^\mu V^* - \lambda(V\, V^* - \frac{g^2}{8\pi^2})^2 + \frac{M^2}{4}\{V^2 + (V^*)^2\}. \tag{13}$$

The vortex selfcoupling in this effective Lagrangian can be determined as [9]

$$\lambda = \frac{2\pi^2 M_W^2}{g^2}. \tag{14}$$

Note that as a low energy Lagrangian, eq.(13) is indeed consistent with eq.(7). At weak gauge coupling the quartic coupling λ is very large. In this nonlinear σ-model limit the radius of the field V is therefore frozen to its expectation value. The only relevant degree of freedom is the phase

$$V(x) = \frac{g}{\sqrt{8}\pi} e^{i\chi}. \tag{15}$$

Substituting this into eq.(13) one indeed obtains precisely eq.(7) with the identification $\phi = 2\chi$. It was shown in [4] that this identification is indeed the correct one, and the factor 2 follows from the fact that the magnetic charge of the monopole is twice the fundamental value relative to the electric charge of the elementary charged excitation $W : g_M g_e = 4\pi$.

The main difference between eq.(13) and eq.(7) is in the description of the charged sector. As explained in [4], the charged particles are solitons of the field V with unit winding number. In the nonlinear σ-model limit those are just vortices of the phase χ. Polyakov's effective Lagrangian eq.(7) does not allow such vortex configurations. In the vortex configuration the field χ (and ϕ) is discontinuous along an infinite cut. The cut contributes to the kinetic energy term in eq.(7), and this contribution is both ultraviolet and infrared divergent. The ultraviolet divergence is of no importance by itself, since the theory is defined with the intrinsic UV cutoff M_W. However the infrared divergence indisputably puts description of the charged states beyond eq.(7). On the other hand eq.(13) recognizes the fact that the field χ is a phase, and therefore 2π discontinuities in it are unphysical and do not cost energy. Thus vortex configurations in eq.(13) are allowed.

Although they are confined dynamically with a linear potential [4], the string tension for this potential is a dynamical effect proportional to $\xi^{1/2}$ and has nothing to do with the extraneous cut contribution.

In fact, if anything the Lagrangian eq.(13) underestimates the energy of such a vortex. The core energy of the vortex should be equal to the mass of the charged vector boson M_W, since this is the lightest charged excitation in the theory. On the other hand the core mass of a vortex in the Lagrangian eq.(13) is given by the UV contribution of the Coulomb potential. With the cutoff of order M_W this is of order $g^2 \ln M_W/g^2$. This is not surprising, since the mass of the W boson indeed comes from the distances of order of its Compton wave length, and is thus well inside the UV cutoff of the effective theory. The situation can be improved by adding to the effective Lagrangian a higher derivative term of the Skyrme type

$$\delta L = \Lambda(\epsilon_{\mu\nu\lambda}\partial_\nu V^* \partial_\lambda V)^2 \tag{16}$$

with

$$\Lambda \propto \frac{1}{g^4}\frac{1}{M_W} \tag{17}$$

With this extra term the core energy of the vortex indeed becomes of the order of M_W.

As we will see in the following, the correct description of the charged sector is crucial for the understanding of the phase transition[5].

3 Deconfining phase transition - The Rough Guide

In this section we give a naive discussion of the expected nature of the deconfining phase transition based on the monopole versus vortex description of the low energy theory.

[5] We note that this is not the only instance in which the charged states play important role. For example as discussed in [6] the presence of these states leads to the breaking of the string of charge two, which in the Polyakov's effective Lagrnagian eq.)7 is strictly stable.

3.1 The monopole binding

Since at zero temperature the monopole contributions are the only relevant ones for confinement, one may be tempted to assume that also at finite, low enough temperature all other effects are not important. The expected value of the transition temperature, as we shall see in a moment is of order g^2, which is much smaller than any mass scale in the theory except for the photon mass. One could therefore start with the working hypothesis that the single self-interacting photon field (or equivalently the monopole ensemble) should be a valid description of the phase transition region. This is the point of view taken in [2], and we now briefly describe the conclusions it leads to.

The first thing to note is that at finite temperature the interaction between monopoles is logarithmic at large distances. The reason is that the finite temperature path integral is formulated with periodic boundary conditions in the Euclidean temperature direction. The field lines are therefore prevented from crossing the boundary in this direction. The magnetic field lines emanating from the monopole have to bend close to the boundary and go parallel to it. So effectively the whole magnetic flux is squeezed into two dimensions. Qualitatively the situation is shown in figure 1.

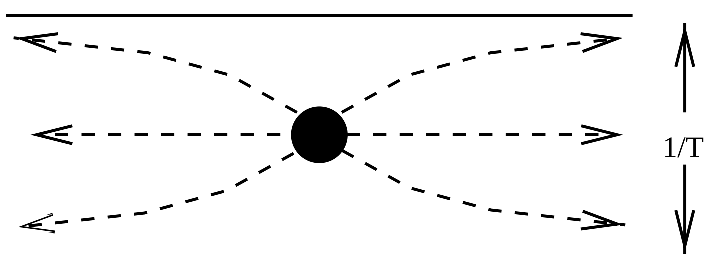

Figure 1: The field of a Monopole-Instanton at finite temperature.

The length of the time direction is $\beta = 1/T$, and thus clearly the field profile is two dimensional on distance scales larger than β. Two monopoles separated by a distance larger than β therefore interact via a two dimensional rather than a three dimensional Coulomb potential, which is logarithmic at large distances. Since the density of the monopoles is tiny $\rho_M \propto \xi$, already at extremely low temperatures $T \propto \xi^{1/2}$ the monopole gas becomes two dimensional. The strength of the logarithmic interaction is easily calculated. The magnetic flux of the monopole should spread evenly in the compact direction once we are far enough from the monopole core The field strength should have only components parallel to the spatial directions. Since the total flux of the monopole is $2\pi/g$, the field strength far from the core is $\tilde{F}_i = \frac{T}{g}\frac{x_i}{x^2}$, and thus the strength of the infrared logarithmic interaction is T^2/g^2.

It is well known [7] that the two dimensional Coulomb gas undergoes the BKT phase transition. In the usual Coulomb gas, where the strength of the interaction λ does not depend on temperature, the particles are bound in pairs at low temperature and unbind at high temperature $T > T_{BKT} = 2\pi\lambda$, where the entropy overcomes the energy. This is the standard BKT phase transition [7] which determines the universality class of $U(1)$ symmetry restoration in 2 dimensions. In the present case the situation is reversed, since the strength of the interaction itself depends on the temperature. At low temperature the interaction is weak, and therefore the particles (monopoles) are free. As the temperature grows, the interaction becomes stronger until at $T_{BKT} = 2\pi\frac{T_{BKT}^2}{g^2}$ the energy overcomes the entropy. Above this temperature the monopoles bind into neutral pairs. Thus based on this simple picture we expect the theory to undergo a BKT phase transition at

$$T_{BKT} = \frac{g^2}{2\pi} \tag{18}$$

Below this temperature the photon should be massive, while above this temperature it should be massless since the cosine term in the Lagrangian eq.(7) is irrelevant.

This is precisely the logic followed in [2]. One can go further and perform more quantitative calculations using the dimensionally reduced Polyakov effective Lagrangian. Dimensional reduction should be perfectly valid in this theory since the critical temperature is much larger than the photon mass. Thus one can consider the two dimensional sine-Gordon theory

$$\mathcal{L} = \frac{g^2}{32\pi^2 T}(\partial_i\phi)^2 + \frac{M^2 g^2}{16\pi^2 T}\cos\phi. \tag{19}$$

This theory, in full agreement with the previous discussion has a BKT phase transition at the temperature given by eq.(18), above which the "dual photon" field ϕ becomes massless.

Moreover, since the 2-d sine-Gordon theory is exactly solvable, one can calculate various correlation functions. In particular the authors of [2] calculated the string tension, and found that it exhibits quite strange behavior. At low temperatures it follows an expected pattern - that is, it decreases with temperature. It becomes extremely small at temperature much smaller than T_{BKT}; however just before T_{BKT} it starts rising sharply and at T_{BKT} actually diverges.

Although the logic leading to the previous discussion seems quite sound, there are several puzzles that arise regarding the results.

• The photon becomes massless in the high temperature phase. In other words, the correlation length in some physical (gauge invariant) channel becomes infinite at high temperature. This contradicts our physical intuition, since one expects that at high temperature any physical system described by a local field theory should become maximally disordered with zero correlation length.

• The phase transition is of BKT type, and therefore belongs to the $U(1)$ universality class. On the other hand we know that the global symmetry that is restored at the phase transition is the magnetic Z_2 [8], and we expect the universality class to be 2D Ising.

• The divergence of the string tension at the critical point looks unphysical.

• Finally, as we explained in the previous section, the effective theory eq.(7) does not allow charged states. It is then difficult to understand in what sense the high temperature phase can be viewed as deconfined.

3.2 Magnetic symmetry restoration and the charged plasma

Let us now take eq. (13) as our starting point. What does one expect from the phase transition in a simple scalar theory of this type? Firstly, clearly one expects the critical temperature to be of order of the expectation value of the scalar field, and therefore parametrically of order g^2. At finite temperature one expects the generation of a positive thermal mass proportional to the temperature and to the coupling constant. Thus the thermal contribution to the effective potential should be of the form

$$\delta_T L = x\lambda T V^* V \tag{20}$$

At $T = g^2/4\pi^2 x$ the total mass term becomes positive, the VEV of V vanishes and the phase transition occurs. One can estimate this temperature more precisely using the following simple argument. Let us neglect for now the monopole induced term. Then we are dealing with the XY model at finite temperature. The dimensional reduction should again be a valid approximation, and thus essentially we have to analyze a 2D model. Now a 2D XY model can be mapped into a sine-Gordon theory of a dual field $\tilde{\chi}$. Performing this dual transformation[6] , we find the Lagrangian

$$L = \frac{T}{2g^2}(\partial_i \tilde{\chi})^2 + \mu \cos \tilde{\chi} \tag{21}$$

where μ is the fugacity of the vortices in the XY model. This sine-Gordon theory has a phase transition at

$$T_{XY} = \frac{g^2}{8\pi}. \tag{22}$$

Therefore we may expect the magnetic symmetry restoring phase transition at T_{XY}. Although parametrically this temperature is of the same order as the BKT transition temperature discussed in the previous subsection, it is four times lower. Another important difference is that the nature of the phase transition is in fact completely different. The phase transition in the model eq. (21) is due to the unbinding of vortices of the field V. Above T_{XY} the vortices are in the plasma phase. These vortices, are as discussed earlier precisely the charged $W^\pm$ bosons of the original Georgi-Glashow model. Thus this phase transition is just what one would naturally call the deconfining phase transition and the vacuum above T_{XY} is the charged plasma.

Note that in this discussion we have neglected completely the effect of monopoles, that is the last term in the Lagrangian eq. (13). Interestingly enough we see that the plasma

[6]To fix the normalization of the kinetic term we should bear in mind that a vortex corresponds to a 2π jump in the field $\chi = 2\phi$ rather than the field ϕ in eq.(7). See also the discussion following eq. (15).

phase is reached at a temperature which is much lower than the T_{BKT} discussed earlier, and thus the monopole binding is irrelevant for the dynamics of the deconfinement. This is not to say that the presence of the monopoles is irrelevant altogether. Clearly, omitting the monopole induced term we enlarged the symmetry of the system from Z_2 to $U(1)$. Hence the effective XY model description and the $U(1)$ universality class predicted by eq.(21). The analysis as it stands is correct for noncompact electrodynamics with charged matter, but not for the Georgi-Glashow model. Next we remedy this problem.

The effect of the monopoles can be qualitatively understood in a simple way. Their presence leads to a confining potential between the charges, that is a linear interaction between the XY model vortices. Thus the monopoles suppress the variation of the phase of the field V except inside the confining string, the width of which is given by the inverse photon mass $d = 1/M \propto \xi^{1/2}$. Whether the presence of the monopoles is important at the "would be" phase transition point T_{XY} depends crucially on the density of charged particles. If the density of the charged particles is very high, so that the average distance between them is smaller than d, the presence of the monopoles is immaterial since the phase of V is disordered already on the short distance scale. However if the density of charges is low, so that the distance between them is larger than d, the presence of the monopoles will suppress the phase transition. In this case at T_{XY} the vacuum will not be disordered, but will look like a dilute gas of charged particles with strings between them. The actual phase transition will then occur at a higher temperature, where the average distance between the charges equals the inverse photon mass.

Interestingly enough in the present case this temperature turns out to be twice the value of T_{XY} and therefore still much lower than T_{BKT}. We will show this rigorously in the next section. Qualitatively though this is easy to understand. The density of charges is proportional to their fugacity

$$\mu \propto e^{\frac{-M_W}{T}} \tag{23}$$

This should be compared to the square of the photon mass, which is given by the zero temperature monopole fugacity. In a theory with very heavy Higgs

$$\xi \propto e^{\frac{-4\pi M_W}{g^2}} \tag{24}$$

The two are equal (up to subleading corrections) at

$$T_{GG} = \frac{g^2}{4\pi} \tag{25}$$

More generally, when the Higgs is not infinitely heavy, the monopole fugacity is smaller and thus the transition temperature will be lower

$$T_c = \frac{g^2}{4\pi\epsilon(\frac{M_H}{M_W})} \tag{26}$$

For lighter Higgs, T_c gets closer to T_{XY}, but is always greater so that $T_{XY} < T_c < T_{BKT}$.

In the next section we will present a more complete analysis based on the renormalization group and exact bosonization. This analysis confirms the simple picture presented here. Thus all the puzzles raised in the previous subsection simply disappear.

• The photon never becomes massless. Even without the monopoles in the plasma phase it acquires the Debye mass given by the cosine term in eq. (21). This mass *rises* with temperature, and thus the physical correlation length decreases.

• Since the monopole term is still relevant at the phase transition, the universality class must be Z_2. We will see this explicitly in the next section.

• The analysis of the previous subsection is only valid below T_{XY}, and thus the divergence of the string tension at $T_{BKT} > T_{XY}$ has nothing to do with the physics of the Georgi Glashow model.

• Finally, the phase transition is driven primarily by the unbinding of charged particles and thus indeed has all the flavor of a deconfinement transition.

Let us now turn to a more quantitative analysis of what is stated above.

4 Deconfining phase transition - renormalization group analysis

For a more formal analysis of the phase transition we find it convenient to use the sine-Gordon formulation in terms of the phase field χ modified to take explicitly into account the finite probability of the appearance of vortices. This Lagrangian has the form

$$\mathcal{L} = \frac{g^2}{8\pi^2 T}(\partial_\mu \chi)^2 + \zeta \cos 2\chi + \mu \cos \tilde{\chi} \tag{27}$$

where ζ is related to the monopole fugacity $\zeta = \xi/T$ and $\tilde{\chi}$ is the field dual to χ,

$$i\partial_\mu \tilde{\chi} = \frac{g^2}{2\pi T}\epsilon_{\mu\nu}\partial^\nu \chi. \tag{28}$$

4.1 The Lagrangian

The way to derive this Lagrangian is the following. The partition function of the sine-Gordon model in the presence of one vortex is

$$Z(x) = \int D[\chi] \exp\{-\int d^2y \frac{g^2}{8\pi^2 T}(\partial_\mu \chi - j_\mu(y,x))^2 + \zeta \cos 2\chi\} \tag{29}$$

The "external current" is

$$j_\mu(y,x) = 2\pi n_\mu(y)\delta(y \in C) \tag{30}$$

with C a curve that starts at the location of the vortex (the point x), and goes to infinity, and n_μ is the unit normal to this curve. The insertion of this current forces the derivative of χ to have a discontinuity across the curve C, so that χ jumps by 2π. This forces χ to have one unit of vorticity concentrated at the point x. Note that even though j_μ explicitly depends on the curve C, the partition function itself does not, since changing the integration variable

$$\chi(x) \to \chi(x) + 2\pi, \qquad x \in S \tag{31}$$

where the boundary of S is $C - C'$ is equivalent to changing C into C' in the definition of the current. The extra linear term in the exponential in (eq.29) is

$$i\tilde{\chi} = \frac{g^2}{2\pi T}\int_C dx_\mu \epsilon_{\mu\nu}\partial_\nu \chi. \tag{32}$$

which is equivalent to (eq.28). An antivortex at y obviously is created by $-j_\mu$. To create several vortices one just inserts an external current which is the sum of the currents which create individual vortices.

A dilute ensemble of vortices and antivortices with (small) fugacity μ is then given by

$$Z=\sum_{n,m}\frac{1}{n!}\frac{1}{m!}\mu^{n+m}\int\Pi_i dx_i\Pi_j dy_j Z(x_i,y_j) \tag{33}$$

The summation over the number of vortices and antivortices can be easily performed leading to the partition function with the Lagrangian eq. (27). The constant μ is the vortex fugacity scaled by the effective UV cutoff imposed on the integration over the coordinates. The vortex fugacity of course is none other but the fugacity of the charged W,

$$\mu=a^{-2}e^{\frac{-M_W}{T}}. \tag{34}$$

The cutoff a is related to the Compton wave length of the W boson, but a more careful determination of it should take into account the fact that in the process of dimensional reduction all modes with frequencies above T have been integrated out. We will not attempt the determination of a, but only note that it is some combination of the scales M_W and $T\propto g^2$, and as such its value always plays a role secondary to the exponential factor of fugacity[7].

An alternative way to derive the Lagrangian eq. (27) is to start directly from the effective theory eq. (13) with the extra Skyrme term eq. (16). In the nonlinear σ-model limit one can cleanly separate the phase of the field V into a smooth part χ and the vortex contribution. The Skyrme term then is proportional to the energy of one vortex, and with the Skyrme coupling chosen the way we discussed in Section 2 is just equal to M_W for a one vortex configuration. The dilute vortex gas approximation then reproduces again the Lagrangian eq. (27).

4.2 The renormalization group equations

Our starting point for the discussion of this section is eq. (27). Since both ξ and μ are small, the importance of different terms in the Lagrangian is determined by their respective

[7]For the same reason the effective value of the monopole induced mass ζ also changes in the dimensionally reduced theory. This change has been calculated in [2]. Again we will not dwell on this finite "renormalization" since this effect is subleading in the same sense.

conformal dimensions calculated in the free theory. The total conformal dimensions of the operators $:\cos 2\chi:$, $:\cos\tilde{\chi}:$ respectively are,

$$\Delta_\xi = \frac{4\pi T}{g^2}, \qquad \Delta_\mu = \frac{g^2}{4\pi T}. \tag{35}$$

Thus at low temperature the charges are irrelevant ($\Delta_\mu > 2$) and the monopoles are relevant ($\Delta_\xi < 2$) and the theory reduces to the sine-Gordon model. At the temperature $T = g^2/4\pi$ the two operators become equally relevant since their conformal dimensions are equal

$$\Delta_\mu = \Delta_\xi = 1. \tag{36}$$

The phase transition point can be determined from the structure of the fixed points of the renormalization group equations. The critical point is the IR unstable fixed point of the RG flow. The renormalization group equations of the sine-Gordon theory have been studied perturbatively in the literature both in the absence [10] and in the presence of the vortices [11]. In terms of the dimensionless parameters

$$t = \frac{4\pi}{g^2}T, \qquad \tilde{\mu} = \mu a^2, \qquad \tilde{\zeta} = \zeta a^2 \tag{37}$$

the lowest order RG equations read

$$\frac{dt}{d\lambda} = \pi^2(\tilde{\mu}^2 - t^2\tilde{\zeta}^2), \tag{38}$$

$$\frac{d\tilde{\mu}}{d\lambda} = (2 - \frac{1}{t})\ \tilde{\mu}, \tag{39}$$

$$\frac{d\tilde{\zeta}}{d\lambda} = (2 - t)\ \tilde{\zeta}. \tag{40}$$

The fixed point structure of these equations is simple.

1. The point T_0

$$t = 0, \qquad \tilde{\mu} = 0, \qquad \tilde{\zeta} = \infty \tag{41}$$

is clearly the zero temperature fixed point. Here the long distance physics is dominated by the monopole induced mass term.

2. The point T_∞

$$t = \infty, \qquad \tilde{\mu} = \infty, \qquad \tilde{\zeta} = 0 \tag{42}$$

is the high temperature fixed point. Here the infrared properties are detemined by the charged plasma effects.

3. The point T_{GG}

$$t = 1, \qquad \tilde{\mu} = \tilde{\zeta}, \qquad \tilde{\mu} = \infty \tag{43}$$

This is the IR unstable fixed point. This is precisely the critical point that corresponds to the deconfining phase transition. At this point both the monopole and the charge plasma induced terms are equally important.

Note that the fixed line $t > 2$, $\tilde{\zeta} = 0$ of massless theories which corresponds to the "confined monopole plasma" is not present, since for $t > 2$ the charged plasma induced mass term is strongly relevant. The same is true for the would be fixed line $t < 1/2$, $\tilde{\mu} = 0$, which is present in the absence of the monopoles and describes the low temperature massless phase of noncompact electrodynamics.

It is instructive to see how the RG equations formalize our qualitative arguments of the previous section. In particular they make clear the role of the point $t = 1/2$ which in the absence of monopoles would be the point where the charged induced term becomes relevant. If one starts the evolution from the initial condition $t = 1/2$, $\tilde{\mu} \gg \tilde{\zeta}$, the running temperature will increase, and $\tilde{\mu}$ will grow into the infrared, while $\tilde{\zeta}$ will grow for a while until the running temperature reaches two. From this point on $\tilde{\zeta}$ will decrease and the system will approach the high temperature fixed point. This corresponds to the situation where at the "would be" critical point $t = 1/2$ the density of charged plasma is so high that the mean distance between the particles is smaller than the width of the confining string.

On the other hand if the initial condition is $\tilde{\mu} < \tilde{\zeta}$, the temperature starts decreasing, making the coupling $\tilde{\mu}$ immediately irrelevant. Thus $\tilde{\mu}$ monotonically decreases to zero and practically does not affect the flow of the other couplings which steadily flow to the zero temperature fixed point. This is the situation where at $t = 1/2$ the density of charged plasma is low, which is indeed true in our model.

Interestingly enough, at $t = 1$ the initial conditions in our model in the case of very heavy Higgs are such that the RG flow starts almost exactly in the region of attraction of the fixed point T_{GG}. At this value of temperature the monopole and charge fugacities are equal. The only difference between $\tilde{\mu}$ and $\tilde{\zeta}$ then is in the prefactors, which are possibly different combinations of M_W and g^2. If $\tilde{\mu}$ and $\tilde{\zeta}$ were exactly equal, the only thing that happens along the flow is that their values grow, but t does not change. Due to the small difference their values are equal at a temperature which slightly differs from 1

$$\frac{M_W}{T} = \frac{4\pi M_W}{g^2} + O(1), \qquad T = \frac{g^2}{4\pi}[1 + O(\frac{g^2}{M_W})] \tag{44}$$

Thus the initial temperature for which the system is in the region of attraction of T_{GG} is slightly different. We conclude that the critical temperature of the Georgi-Glashow model is indeed, up to corrections of order g^2/M_W, given by $T_{GG} = \frac{g^2}{4\pi}$.

A comment is in order about our use of the perturbative renormalization group equations. As written the equations (40) are only valid for small $\tilde{\mu}$ and $\tilde{\zeta}$. The initial values of these constants in our model are indeed small, and thus the flow is well described by these equations for quite a while. The fixed points however correspond to large values of at least some parameters, and thus quantitative details of the flow in this region of parameter space are different. However the existence of the three fixed points T_0, T_∞ and T_{GG} is an exact statement. For T_0 and T_∞ this is a consequence of the fact that one of the couplings vanishes ($\tilde{\mu}$ or $\tilde{\zeta}$) and thus these fixed points are the same as in the corresponding sine-Gordon theory. The existence of T_{GG} is a consequence of the exact duality in the model eq. (27). The transformation $2\chi \to \tilde{\chi}$ in the path integral leads to the same partition function but with new parameters as follows $t \to 1/t$, $\tilde{\mu} \to \tilde{\zeta}$, $\tilde{\zeta} \to \tilde{\mu}$. This symmetry is nothing but the famous T-duality (see for example [12] and references therein). The point T_{GG} is the fixed point of this transformation and is thus guaranteed also to be a fixed point of the exact renormalization group equations.

4.3 The universality class of the phase transition

Since the deconfining phase transition is due to the restoration of Z_2 symmetry we expect that it is in the 2-dimensional Ising universality class. This can be shown exactly by studying the theory eq. (27) at the fixed point $t = 1$. The following discussion follows closely [13][See also chapter 21 in [14]].

The theory described by eq. (27) can be fermionized by using the standard bosonization/fermionization techniques. Since at $t = 1$ both cosine terms have dimension 1, the resulting fermionic theory is a theory of free massive fermions. In our notations, the Dirac fermionic field is defined as

$$\psi_R = a^{-1/2} i \exp[i(\chi + \frac{\tilde{\chi}}{2})], \qquad \psi_L = a^{-1/2} \exp[-i(\chi - \frac{\tilde{\chi}}{2})], . \tag{45}$$

The kinetic term in eq. (27) then becomes the kinetic term of the field ψ, while the cosine terms become

$$\begin{aligned} a^{-1} \cos 2\chi &= i[\psi_R^\dagger \psi_L - H.c.] \\ a^{-1} \cos \tilde{\chi} &= i[\psi_R^\dagger \psi_L^\dagger - H.c.]. \end{aligned} \tag{46}$$

Thus the mass term in the fermionized Lagrangian is diagonalized by introducing the real Majorana fermions

$$\rho = \frac{\psi + \psi^\dagger}{\sqrt{2}}, \quad \sigma = \frac{\psi - \psi^\dagger}{i\sqrt{2}}. \tag{47}$$

The mass of the fermion ρ is $\mu a + \zeta a$, while the mass of σ is $\mu a - \zeta a$. For $\mu = \zeta$ the model contains one massive and one massless fermion. At the fixed point $\tilde{\mu} \to \infty$ the massive fermion decouples. Thus at the point T_{GG} the theory is that of one massless Majorana fermion. It is well know that this theory precisely describes the critical point of the 2D Ising model.

Thus we indeed see that the phase transition is in the universality class of the 2D Ising model.

5 From the magnetic vortices to the Polyakov line

It is standard practice to study the high temperature phase of gauge field theories using the effective action in terms of the zeroth component of the vector potential A_0. In this section we want to relate our analysis which was performed for the phase of the vortex operator χ to this standard procedure.

The relation in fact is not difficult to make. The free energy of the charged $W^\pm$ bosons in the Lagrangian eq. (27) is given by the term $\cos\tilde{\chi}$. On the other hand this same free energy in the standard calculation is represented by the insertion of the Polyakov line with charge two. We thus identify the Polykov line with the exponential of the dual field

$$P = \exp\{\frac{i}{2}\tilde{\chi}\} \tag{48}$$

and the dual field $\tilde{\chi}$ with the Abelian vector potential

$$\tilde{\chi} = 2g\beta A_0 \tag{49}$$

Up to the monopole induced term, the Lagrangian eq. (27) is equivalent to the sine-Gordon theory of the field $\tilde{\chi}$ in eq. (21). With the identification in eq. (49), this is

$$\frac{2}{T}(\partial_i A_0)^2 + \mu\cos(\frac{2g}{T}A_0) \tag{50}$$

The monopole induced term also can be written in terms of the vector potential. Its form in terms of the Polyakov line P is precisely the same as the form of the charge plasma induced term in terms of the vortex field V. Recall that the origin of the plasma induced term $\cos\chi$ in eq. (27) is the dimensionally reduced "Skyrme term" of eq. (16). Thus just like we derived the sine Gordon Lagrangian eq.(27) starting from the effective Lagrangian for the vortex field, we can follow the same steps backwards but this time expressing everything in terms of P. Due to this duality between V and P we conclude that the monopole induced term is the dimensionally reduced Skyrme term for the Polyakov line. The full effective Lagrangian for the vector potential is then

$$L = \frac{2}{T}(\partial_i A_0)^2 + \mu\cos(\frac{2g}{T}A_0) + \frac{a^2}{4\pi^2}\ln\left(\tilde{\zeta}\right)(\epsilon_{ij}\partial_i P\partial_j P^*)^2 \tag{51}$$

The monopole in this Lagrangian is represented by a vortex of the field $\frac{g}{T}A_0$ with unit vorticity. The coefficient of the Skyrme term in eq. (51) is such that the action of such a unit vortex is equal to the action of the "core" of the monopole.

The cosine term in this expression is the potential for A_0 induced by the nonvanishing density of the charged particles in the thermal ensemble. Naturally, it contains the Debye "electric" mass term for A_0 and also higher interactions. Note that as opposed to strongly interacting theories, where similar effective Lagrangians have been derived only in the derivative expansion, the Lagrangian eq. (51) is valid on all distance scales longer than $1/T$. Thus in principle it can be used to calculate correlation functions in a large momentum range. However, if one is interested in the long distance behavior of the correlators, at temperatures above T_{GG} the higher derivative Skyrme term can be neglected. This is in accordance with our analysis of the previous section which showed that the monopole induced term is irrelevant above T_{GG}.

An interesting example of a correlation function is the correlator of vortex operators $< V(x)V^*(y) >$. At low temperatures, since the magnetic Z_2 symmetry is spontaneously broken this correlation function tends to a constant at large separations. The corrections to this constant value are given by the correlator of the field χ. At zero temperature the correlator is

$$< V(x)V^*(y) >= \frac{g^2}{8\pi^2}\exp\{-\frac{1}{2} < \chi(x)\chi(y) >\} = \frac{g^2}{8\pi^2}\exp\{-\frac{16\pi^2}{g^2}Y_3(x-y)\} \tag{52}$$

where $Y_3(x-y)$ is the 3d Yukawa potential with the mass M. At low temperature ($T < T_{GG}$) the infrared asymptotics (at distances $|x-y| > 1/T$) is instead

$$< V(x)V^*(y) >= \frac{g^2}{8\pi^2}\exp\{-\frac{16\pi^2 T}{g^2}Y_2(x-y)\} \tag{53}$$

with $Y_2(x-y)$ - the 2D Yukawa potential with the mass which includes the effects of integrating out the nonzero Matsubara modes, as calculated in [2]. This expression follows from eq. (27) neglecting the $\cos\tilde{\chi}$ term, which is indeed negligible in the infrared.

At high temperatures $T > T_{GG}$ instead it is convenient to use eq. (51) with the omission of the third term[8]. The calculation of the correlator of the vortex operators proceeds along the lines of [15, 8]. The insertion of $V(x)$ and $V^*(y)$ creates the Z_2 domain wall stretching between the points x and y. In terms of the sine-Gordon theory eq. (51) this domain wall is just the kink, and thus the domain wall tension is equal to the soliton mass M_s [16]. The correlator then is

$$< V(x)V^*(y) >= \exp\{-M_s|x-y|\} \tag{54}$$

with the soliton mass reading as

$$M_s = a^{-1}\frac{2\Gamma(p/2)}{\sqrt{\pi}\Gamma(\frac{p+1}{2})}\left[\frac{\pi\Gamma(\frac{1}{p+1})}{\Gamma(\frac{p}{p+1})}\,2\tilde{\mu}\right]^{\frac{p+1}{2}}, \tag{55}$$

where

$$p = \frac{g^2}{8\pi T - g^2}. \tag{56}$$

Thus we find that the correlation function of the vortex operators decreases exponentially in the high temperature phase as it should in the phase with restored symmetry. Note that were we to neglect the Debye mass term we would have found that below T_{BKT} the correlator tends to a constant at infinity, while at $T > T_{BKT}$ it does decay at large distances but only as a power, since the mass of the soliton would vanish.

We note that the result eq. (54) implies that in the deconfining phase the monopoles are confined linearly. We remind the reader that insertion of an operator V is equivalent to creation of a monopole[9]. Thus the logarithm of the correlation function can be interpreted directly as the intermonopole potential. Indeed at zero temperature one can read off the intermonopole potential from eq.(52). The potential is the Debye screened 3D Coulomb potential. At nonzero temperature eq. (53) gives it as the 2D Coulomb potential at short

[8]Our calculation is only valid in the range of temperatures below the Higgs expectation value h. At $T > h$ the theory becomes essentially nonabelian, since the Higgs particle as well as W bosons are present in the "vacuum" in large numbers, and our methods become inapplicable. In fact in this range there may be another phase transition corresponding to the "melting" of the Higgs expectation value. Our analysis in this and previous sections has nothing to say about this regime.

[9]Strictly speaking V creates a Z_2 monopole - that is the monopole with half the magnetic charge than the 'tHooft-Polyakov solution. However the interaction between these objects is qualitatively the same.

distances, again screened at distances $x^2 > 1/\zeta$. At high temperature then we see from eq. (54) that the interaction between the monopoles is linear with the "string tension" equal to M_s. Thus the behaviour of the monopoles is in many senses "dual" to that of electric charges[10].

6 Conclusions

In this paper we have studied the deconfining phase transition in the 2+1 dimensional Georgi Glashow model. This model to this day remains the only analytically solvable confining theory, and it is therefore interesting to see how the deconfining transition occurs in this simple yet nontrivial setting.

We found that the phase transition is indeed associated with the restoration of the magnetic Z_2 symmetry with the universality class of 2D Ising model. We have shown that the transition is driven by the appearance of the charged plasma rather than by the binding of the monopoles. The presence of the monopoles is significant, since it pushes the value of the critical temperature up by a factor of two (in the infinitely heavy Higgs limit) relative to the plasma transition in the noncompact electrodynamics. We have derived the effective action in the high temperature phase without recourse to the derivative expansion and have calculated the correlation function of the vortex operators. These correlators decay exponentially above the phase transition, which is consistent with the restoration of the magnetic symmetry. This also shows that the monopoles are bound linearly at high temperatures due the charged plasma effects, and these effects are much more important than the monopole self interaction, which by itself leads only to a logarithmic intermonopole potential.

We believe that these qualitative features are universal and should generalize not only to the strongly interacting non Abelian theories in 2+1 dimensions, but also in large measure to 3+1 dimensional QCD along the lines discussed in [8]. Unfortunately we are not yet able to generalize the present quantitative analysis to these more interesting theories.

[10] Of course one always has to keep in mind that physically monopoles and charges are very different objects in this models: the charges are particles, while the monopoles are instantons.

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